3-equivalent 4-tuples with 4 diagonal entries

Walter Trump, 2018-03-05

The trimagic square 8a has a pair of 3-equivalent 4-tuples with the diagonal entries 20, 29, 116 and 125. When the numbers of the tuples are swapped both diagonals are no longer trimagic, because the above mentioned numbers are replaced by 43, 70, 75 and 102. But the other numbers of column 1 and 3, which you can see in the yellow cells, can be interchanged. They build a pair of 3-equivalent 8-tuples.

8a with a pair of 3-equivalent 4-tuples

20	58	43	85	68	138	93	5	91	134	108	27
21	26	127	143	74	78	56	49	97	84	112	3
70	111	29	104	114	139	55	53	4	24	117	50
35	36	63	40	133	101	1	123	38	76	98	126
57	132	131	72	42	83	122	25	17	10	100	79
130	113	39	16	9	99	30	64	94	59	137	80
15	32	106	129	136	46	115	81	51	86	8	65
88	13	14	73	103	62	23	120	128	135	45	66
110	109	82	105	12	44	144	22	107	69	47	19
75	34	116	41	31	6	90	92	141	121	28	95
124	119	18	2	71	67	89	96	48	61	33	142
125	87	102	60	77	7	52	140	54	11	37	118

8b with interchanged entries of yellow cells

20	58	43	85	68	138	93	5	91	134	108	27
127	26	21	143	74	78	56	49	97	84	112	3
70	111	29	104	114	139	55	53	4	24	117	50
63	36	35	40	133	101	1	123	38	76	98	126
131	132	57	72	42	83	122	25	17	10	100	79
39	113	130	16	9	99	30	64	94	59	137	80
106	32	15	129	136	46	115	81	51	86	8	65
14	13	88	73	103	62	23	120	128	135	45	66
82	109	110	105	12	44	144	22	107	69	47	19
75	34	116	41	31	6	90	92	141	121	28	95
18	119	124	2	71	67	89	96	48	61	33	142
125	87	102	60	77	7	52	140	54	11	37	118

Aale de Winkel suggested to search for such squares. He also thought whether it would be possible to correct the diagonals after swapping the 4-tuples.
In fact this can be done if the tuples are in column h and column k with $h + k = 13$.
This means the tuples have to be arranged symmetrically with respect to the vertical symmetry axis of the square.
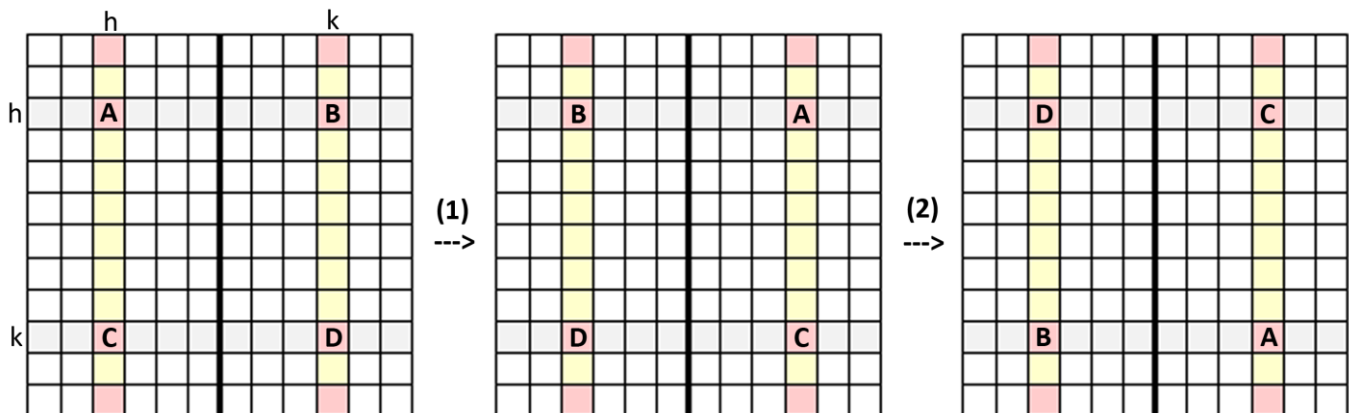
In step (1) the 4-tuples of column h and k are swapped. Now the diagonals are not trimagic. In step (2) the rows h and k are swapped. The diagonals are trimagic again. Unfortunately the obtained square is not essentially different from the square which can be achieved by swaping the yellow 8-tuples.